SAMPLING A LIQUID FROM A DEFORMABLE STRATUM THROUGH A HIGHLY PERMEABLE WINDOW

A. V. Kolmogorov and V. N. Nikolaevskii

UDC 532.546

1. In the axially symmetrical case, the system of equations for an elastoporous stratum saturated with liquid consists [1] of the equations of motion for the solid phase:

$$\nabla^2 u_r - \frac{u_r}{r^2} + \frac{1}{1-2\nu}\frac{\partial e}{\partial r} - \frac{2(1-\nu)}{1-2\nu} g \frac{\partial p}{\partial r} = 0,$$

$$\nabla^2 u_z + \frac{1}{1-2\nu}\frac{\partial e}{\partial z} - \frac{2(1-\nu)}{1-2\nu} g \frac{\partial p}{\partial z} = 0,$$

$$\varepsilon = (1-m_0)\beta_1 K, \quad g = (1-\varepsilon)(1-2\nu)[2G(1-m_0)(1-\nu)]^{-1},$$

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}, \quad e = \frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}; \tag{1.1}$$

the equations of motion for the liquid:

$$\frac{k}{\mu}\frac{\partial p}{\partial r} = -m_0\left(w_r - \frac{\partial u_r}{\partial t}\right), \quad \frac{k}{\mu}\frac{\partial p}{\partial z} = -m_0\left(w_z - \frac{\partial u_z}{\partial t}\right) \tag{1.2}$$

and the equations of continuity for the solid and liquid phases:

$$\frac{\partial m}{\partial t} + \frac{\beta_1}{3}\frac{\partial \theta^f}{\partial t} - \beta_1(1-m_0)\frac{\partial p}{\partial t} - (1-m_0)\frac{\partial e}{\partial t} = 0; \tag{1.3}$$

$$\frac{\partial m}{\partial t} + \beta_2 m_0 \frac{\partial p}{\partial t} - m_0\left(\frac{\partial w_r}{\partial r} + \frac{w_r}{r} + \frac{\partial w_z}{\partial z}\right) = 0; \tag{1.4}$$

$$\theta^f = \sigma^f_{rr} + \sigma^f_{\theta\theta} + \sigma^f_{zz} = 2G(1-m_0)(1+\nu)(1-2\nu)^{-1}e + 3\varepsilon p. \tag{1.5}$$

Here the variables are the deviations from stationary values of the quantities and the symbols are as follows: m, porosity (m_0, the initial value); u_i, a component of the solid-phase displacement; w_i, a component of the liquid velocity; e, the bulk deformation of the solid phase; p, pore pressure; σ_{ij}, the stress in the solid phase; $\sigma^f{}_{ij}$, the effective Tertsagi stress; θ^f, the first invariant of this; k, permeability; μ, viscosity of the liquid; β_1 and β_2, compressibility coefficients for the material of the solid and liquid phases; and K, G, and ν, the bulk modulus, shear modulus, and Poisson's ratio for the matrix.

Let the viscous liquid be taken from a closed stratum of radius R and thickness 2h via a planar horizontal crack of radius ρ with a flow rate Q(t). In the case of highly effective large-scale hydraulic disruption [2], the radius ρ may be comparable with R and much greater than h. For simplicity we assume that the depression in the stratum due to the liquid tapoff does not cause any change in the total stress, or in other words in the rock pressure Γ^0_{ij}, acting on the external edge of the stratum from the surrounding rocks. Correspondingly, at the boundaries of the stratum Γ^0_{ij} =const and the increments in the rock pressure are zero: $\Gamma_{ij} = \sigma^f{}_{ij} - p\delta_{ij} = 0$.

The coordinate system is chosen such that the origin coincides with the center of the crack and the stratum (Fig. 1a). As the boundary conditions are symmetrical, we need consider only the region $z \geqslant 0$, i.e., the problem corresponds to tapping off liquid through a crack in the top or bottom of a stratum of thickness h (Fig. 1b).

2. The boundary conditions for an infinite stratum take the form

$$\Gamma_{zz} = 0, \quad \sigma^f_{rz} = 0, \quad \partial p/\partial z = 0 \quad (z = h,\ 0 \leqslant r < \infty); \tag{2.1}$$

$$\sigma^f_{rz} = 0, \quad u_z = 0 \quad (z = 0,\ 0 \leqslant r < \infty); \tag{2.2}$$

$$\partial p/\partial z = 0 \quad (\rho \leqslant r < \infty,\ z = 0), \quad p = p_*(t) \quad (0 \leqslant r < \rho,\ z = 0). \tag{2.3}$$

Yakutsk and Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 70-77, July-August, 1983. Original article submitted June 8, 1982.

 0021-8944/83/2404-0510$07.50

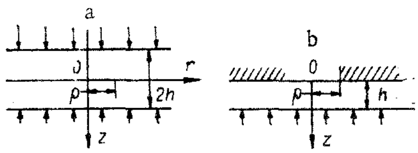

Fig. 1

and the initial conditions are

$$p = p_0, \; u_i = 0, \; \sigma_{ij}^f = 0. \tag{2.4}$$

The variations in pore pressure p over the thickness h of the stratum are less than the characteristic variations over r, and therefore in (1.1) it is permissible to replace p(r, z, t) as an approximation by the weighted mean pressure p*(r, t) taken over the thickness of the stratum:

$$p^*(r, t) = \frac{1}{h}\int_0^h p(r, z, t)\, dz, \quad \partial p^*/\partial z = 0. \tag{2.5}$$

We use integral Hankel transformations [3] of the first order with the first equation of (1.1) and of zero order with the second equation in (1.1). Then we get correspondingly

$$\begin{gathered} \frac{\partial^2 \bar{u}_r}{\partial z^2} - a\xi^2 \bar{u}_r - b\xi \frac{\partial \bar{u}_z}{\partial z} + c\xi \bar{p}^* = 0, \\ a\frac{\partial^2 \bar{u}_z}{\partial z^2} - \xi^2 \bar{u}_z + b\xi \frac{\partial \bar{u}_r}{\partial z} = 0, \end{gathered} \tag{2.6}$$

where $a = 2(1-\nu)/(1-2\nu)$; $b = (1-2\nu)^{-1}$; $c = (1-\varepsilon)/[2G(1-m_0)]$;

$$\bar{u}_r = \int_0^\infty r u_r(r, z, t) J_1(\xi r)\, dr; \quad \bar{u}_z = \int_0^\infty r u_z(r, z, t) J_0(\xi r)\, dr;$$

$$\bar{p}^* = \int_0^\infty r p^*(r, t) J_0(\xi r)\, dr$$

and it is assumed that the following conditions are obeyed:

$$\begin{gathered} r^2\partial u_r/\partial r \to 0, \; r u_r \to 0 \;\; (r \to 0), \\ \sqrt{r}\partial u_r/\partial r \to 0, \; \sqrt{r} u_r \to 0 \;\; (r \to \infty). \end{gathered}$$

The solutions to (2.6) are sought in the form

$$\begin{aligned} \bar{u}_r &= [A(\xi) + \xi z B(\xi)]e^{\xi z}\xi^{-1} + [C(\xi) + \xi z D(\xi)]e^{-\xi z}\xi^{-1} + \bar{p}^*\xi^{-1}, \\ \bar{u}_z &= [-A(\xi) + B(\xi)(3 - 4\nu - \xi z)]e^{\xi z}\xi^{-1} + [C(\xi) + \\ &\quad + (3 - 4\nu + \xi z)D(\xi)]e^{-\xi z}\xi^{-1}. \end{aligned} \tag{2.7}$$

The functions A(ξ), B(ξ), C(ξ), D(ξ) should be selected in accordance with boundary conditions (2.1) and (2.2). If we apply Hankel transformations to (1.2)-(1.5), we get on the basis of (2.7) that

$$\frac{\partial \bar{p}}{\partial t} + \gamma \frac{\partial \bar{p}}{\partial t}\varphi(\xi h)\,\mathrm{ch}(\xi z) + \left(\xi^2 \bar{p} - \frac{\partial^2 \bar{p}}{\partial z^2}\right) k(\alpha\mu)^{-1} = 0, \tag{2.8}$$

where

$$\bar{p} = \int_0^\infty p(r, z, t)\, r J_0(\xi r)\, dr; \quad \varphi(\xi h) = 2\mathrm{sh}(\xi h)/(\mathrm{sh}(2\xi h) + 2\xi h);$$

$$\begin{gathered} \alpha = (1 - m_0)(1 - \beta_1 K)(\beta_1 + g) + m_0(\beta_2 + g); \; \gamma = (1-\varepsilon)^2(1-2\nu)^2/2G \\ \times (1 - m_0)(1 - \nu)\alpha. \end{gathered}$$

If now we integrate (2.8) with respect to z from 0 to h and divide by h, we get an equation for the transform of the mean weighted pressure p*:

$$\frac{\partial \bar{p}^*}{\partial t} - \gamma\psi(\xi h)\frac{\partial \bar{p}^*}{\partial t} + k(\alpha\mu)^{-1}\xi^2\bar{p}^* = k(\alpha\mu)^{-1}\frac{1}{h}\left(\frac{\partial \bar{p}}{\partial z}\right)\Bigg|_0^h, \tag{2.9}$$

where $\psi(\xi h) = 2\sinh^2(\xi h)/[\xi h(\sinh(2\xi h) + 2\xi h)]$.

We use the method of [4] to replace the inhomogeneous boundary condition of (2.3) by a homogeneous one at $z = 0$:

$$\begin{aligned} \partial p/\partial z &= Q(t)\mu/(2\pi k\rho\sqrt{\rho^2 - r^2}) \quad (0 \leqslant r < \rho), \\ \partial p/\partial z &= 0 \quad (\rho \leqslant r < \infty), \end{aligned} \tag{2.10}$$

where $Q(t)$ is the flow rate of the liquid as in (2.3).

We apply a zero-order Hankel transformation to (2.10) and substitute the resulting expression into (2.9). Then we can eliminate the value of the gradient $\partial\bar{p}/\partial z$ and obtain the effective value

$$\frac{\partial \bar{p}^*}{\partial t}(1 + \gamma\psi(\xi h)) + k\xi^2(\alpha\mu)^{-1}\bar{p}^* = Q(t)\sin(\xi\rho)/2\pi\alpha h\rho\xi. \tag{2.11}$$

The solution to (2.11) with $\bar{p}^*(\xi, 0) = 0$ is [5]

$$\int_0^\infty \bar{p}^*(\xi, t)e^{-st}dt = -\mu[2\pi kh\xi\rho)(s\tau(\xi h) + \xi^2)]^{-1}\sin(\xi\rho)\int_0^t Q(t)e^{-st}dt, \tag{2.12}$$

where $\tau(\xi h) = [\mu\alpha(1 + \gamma/2)][k(1 + \Phi(\xi h))]^{-1}$;

$$\Phi(\xi h) = (\gamma/2)[\xi h(\mathrm{sh}(2\xi h) + 2\xi h) - 4\,\mathrm{sh}^2(\xi h)]/[2\gamma\mathrm{sh}^2(\xi h) + \xi h(\mathrm{sh}(2\xi h) + 2\xi h)].$$

In the case of a very thin stratum, $\xi h \to 0$ and $\Phi(\xi h) \to 0$, while for a thick stratum $\xi h \to \infty$, $\Phi(\xi h) \to \gamma/2$.

If a constant flow rate is taken from the crack

$$Q(t) = Q_0 U(t), \quad U(t) = \begin{cases} 0, & t \leqslant 0, \\ 1, & t > 0, \end{cases}$$

then $\tilde{Q}(s) = Q_0 s^{-1}$ and it follows from (2.12) that

$$\int_0^\infty \bar{p}^*(\xi, t)e^{-st}dt = -(\mu Q_0/2\pi kh)\sin(\xi\rho)/(s\xi\rho(s\tau(\xi h) + \xi^2)). \tag{2.13}$$

The distribution of the mean weighted pressure in the stratum is obtained from (2.13) by successively applying inverse Laplace and Hankel transformations:

$$p^*(r, t) = -Q_0\mu/(2\pi kh)\int_0^\infty \sin(\xi\rho)(1 - \exp(-t\xi^2/\tau(\xi h)))J_0(\xi r)\xi^{-2}\rho^{-1}d\xi. \tag{2.14}$$

For a thin stratum we have from (2.14) that

$$p^*(r, t) = -Q_0\mu/(2\pi kh)\int_0^\infty \sin(\xi\rho)(1 - \exp(-t\xi^2\varkappa))J_0(\xi r)(\xi^2\rho)^{-1}d\xi, \tag{2.15}$$

which coincides with the solution in the local-elastic formulation [5] for $\varkappa = \mu/[k\alpha(1 + \gamma/2)]$. If the stratum is an ideally cemented porous medium [1], then $\varepsilon = 1$ and $\gamma = 0$, and from (2.14) we have (2.15) with $\varkappa = \mu/[km_0(\beta_2 - \beta_1)]$. In that case, the effective elastic capacity of the stratum is determined only by the difference in compressibilities between the solid and liquid phases, or in other words the locally elastic solutions are obtained from (2.14) if the stratum is very thin or if the deformation of the stratum is due to the compressibility of the phases, while the displacement of the solid particles one relative to another can be neglected.

If on the other hand the stratum is a soft uncemented rock, then the deformation is due to displacement of the solid particles relative to one another. Then $\varepsilon = 1$, $\gamma \approx 1 - 2\nu$, and therefore

$$\tau(\xi h) = [\mu(1 - 2\nu)(3 - 2\nu)/4kG(1 - m_0)(1 - \nu)][1 + \Phi(\xi h)]^{-1}. \tag{2.16}$$

From (2.7) and (2.14) we get equations for displacements u_r and u_z in the stratum:

$$u_r=\int_0^\infty \bar{p}^*(\xi,t)\{g\varphi(\xi h)[\xi z\,\mathrm{sh}(\xi z)+\mathrm{ch}(\xi z)(1-2\nu-\xi h\,\mathrm{cth}(\xi h))]+1\}\,J_1(\xi r)\,d\xi,$$

$$u_z=\int_0^\infty \bar{p}^*(\xi,t)\,g\varphi(\xi h)\{\mathrm{sh}(\xi z)[2(1-\nu)+\xi h\,\mathrm{cth}(\xi h)] - \xi z\,\mathrm{ch}(\xi z)\}\,J_0(\xi r)\,d\xi.$$

Let the pressure at the crack $p_*(t)$ be equal to the pressure averaged over the area of the crack:

$$p_*(t)=\langle p^*\rangle=\frac{2}{\rho^2}\int_0^\rho rp^*(r,t)\,dr.$$

Then

$$p_*(t)=-\frac{Q_0\mu}{2\pi kh}\int_0^\infty \sin(\xi\rho)(\xi^3\rho^2)^{-1}[1-\exp(-t\xi^2/\tau(\xi h))]\,J_1(\xi\rho)\,d\xi.$$

3. If the stratum is bounded by an impermeable contour of radius R, we can specify the boundary conditions at the contour:

$$u_r=0,\ \sigma_{rz}^f=0,\ \partial p/\partial r=0\ (r=R,\ 0\leqslant z\leqslant h). \tag{3.1}$$

Then to solve system (1.1)-(1.4) while retaining conditions (2.1)-(2.3) we should use integral Hankel transformations over a finite interval [3]:

$$\bar{u}_r(\xi,z,t)=\int_0^R ru_r(r,z,t)\,J_1(\xi r)\,dr,$$
$$\bar{u}_z(\xi,z,t)=\int_0^R ru_z(r,z,t)\,J_0(\xi r)\,dr,\quad \bar{p}^*(\xi,t)=\int_0^R rp^*(r,t)\,J_0(\xi r)\,dr. \tag{3.2}$$

Again in (1.1) we replace the pore pressure p(t, z, r) by the mean weighted value p*(r, t) and integrate (1.1) by parts and use boundary conditions (3.1) to get the equations

$$\frac{\partial^2\bar{u}_r}{\partial z^2}-a\xi^2\bar{u}_z-b\xi\frac{\partial\bar{u}_z}{\partial z}+c\xi\bar{p}^*+\left[a\frac{\partial u_r}{\partial r}+b\frac{\partial u_z}{\partial z}-cp^*\right]_R RJ_1(\xi R)=0,$$
$$a\frac{\partial^2\bar{u}_z}{\partial z^2}-\xi^2\bar{u}_z-b\xi\frac{\partial\bar{u}_r}{\partial z}+[u_z]_R\xi RJ_1(\xi R)=0. \tag{3.3}$$

If we choose ξ in (3.3) such as to be the root of

$$\xi_i R\ J_1(\xi_i R)=0, \tag{3.4}$$

then we use the following equations [3] to invert (3.2):

$$u_r(r,z,t)=\frac{2}{R^2}\sum_i \bar{u}_r(\xi_i,z,t)\frac{J_1(\xi_i r)}{[J_0(\xi_i R)]^2},$$
$$u_z(r,z,t)=\frac{2}{R^2}\sum_i \bar{u}_z(\xi_i,z,t)\frac{J_0(\xi_i r)}{[J_0(\xi_i R)]^2},\quad p^*(r,t)=\frac{2}{R^2}\sum_i \bar{p}^*(\xi_i,t)\frac{J_0(\xi_i r)}{[J_0(\xi_i R)]^2}. \tag{3.5}$$

Here the sum is taken over all positive roots of (3.4). Then (3.3) will have the form of (2.6), and the solutions will take the form of (2.7), but with ξ_i everywhere replacing ξ. Similarly, we get the expression for the mean weighted pore pressure:

$$\bar{p}^*(\xi_i,t)=(-Q_0\mu/(\pi kh))\sin(\xi_i\rho)\left[1-\exp\left(-t\xi_i^2/\tau(\xi_i h)\right)\right]\rho^{-1}\xi_i^{-2}. \tag{3.6}$$

Inversion of (3.6) according to (3.5) gives the solution as

$$p^*(r,t)=(-Q_0\mu/(\pi kh))\,2R^{-2}\sum_i \sin(\xi_i\rho)\left[1-\exp\left(-t\xi_i^2/\tau(\xi_i h)\right)\right]\frac{J_0(\xi_i r)}{[J_0(\xi_i R)]^2}\rho^{-1}\xi_i^{-2}. \tag{3.7}$$

For large values of $\xi_i R$ one can use the asymptotic expansions of the Bessel functions [4]:

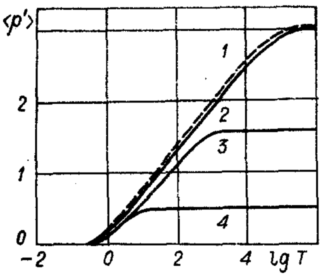

Fig. 2

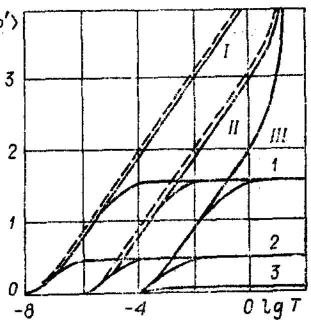

Fig. 3

$$J_0(R\xi_i) \approx \sqrt{2/(\pi R\xi_i)} \cos(R\xi_i - \pi/4),$$
$$J_1(R\xi_i) \approx \sqrt{2/(\pi R\xi_i)} \cos(R\xi_i - 3\pi/4). \tag{3.8}$$

In accordance with (3.4) and (3.8) we have

$$\sqrt{2\xi_i/\pi\rho}\,\sin(\xi_i R - \pi/4) = 0,\ \xi_i R = \pi i - \pi/4,$$
$$\Delta\xi_i = \xi_i - \xi_{i-1} = \pi/R.$$

Then the series of (3.7) can be transformed to the form

$$p^*(r, t) = (-Q_0\mu/\pi kh)\sum_i \sin(\xi_i\rho)\left(1 - e^{-t\xi_i^2/\tau(\xi_i h)}\right)\frac{\pi}{\xi_i^2\rho R} J_0(\xi_i r).$$

For $\xi_i R \to 0$, $\Delta\xi_i \to d\xi_i$ one can replace the sum by an integral:

$$p^*(r, t) = (-Q_0\mu/\pi kh)\lim_{\xi_i R\to\infty}\sum_i \frac{\sin(\xi_i\rho)}{\xi_i^2\rho}\left(1 - e^{-t\xi_i^2/\tau(\xi_i h)}\right) J_0(r\xi_i)\,\Delta\xi_i =$$
$$= (-Q_0\mu/\pi kh)\int_0^\infty \sin(\xi_i\rho)\left(\xi_i^2\rho\right)^{-1}\left(1 - e^{-t\xi_i^2/\tau(\xi_i h)}\right) J_0(r\xi_i)\,d\xi_i,$$

which again leads to the solution of (2.14) for an unbounded stratum. For the displacements we have the expressions

$$u_r = (2/R^2)\sum_i (\bar{p}^*(\xi_i, t)/\xi_i)\{g\varphi(\xi_i h)[\xi_i z\,\mathrm{sh}(\xi_i z) +$$
$$+ \mathrm{ch}(\xi_i z)(1 - 2\nu - \xi_i h\,\mathrm{cth}(\xi_i h))] + 1\} J_1(\xi_i r)/[J_0(R\xi_i)]^2,$$
$$u_z = (2/R^2)\sum_i (\bar{p}^*(\xi_i, t)/\xi_i)\, g\varphi(\xi_i h)\{\mathrm{sh}(\xi_i z)[2(1+\nu) + \xi_i h\,\mathrm{cth}(\xi_i h)]$$
$$- \xi_i z\,\mathrm{ch}(\xi_i z)\} J_0(\xi_i r)/[J_0(R\xi_i)]^2.$$

4. Let the matrix show creep due to the rheological properties of the links between solid particles, with $\varepsilon \ll 1$. In that case the moduli relating to the repacking deformations are time operators. These are differential or integral operators for a linear viscoelastic material [6].

If the type of the boundary conditions does not alter throughout the process at the boundaries of the stratum and at the crack, one can use Volterra's principle [7], viz., the solution to the viscoelastic problem can be found by replacing all elastic constants by Laplace transforms of the corresponding operators in the solution for the analogous problem obtained by integral Laplace transformation, with subsequent reversal of the new solution.

We derive a solution for a viscoelastic stratum by means of Volterra's principle. With the condition $\varepsilon \ll 1$, $\gamma = 1 - 2\nu$, and the function $\tau(\xi h)$ is defined by (2.16). If $\nu = \mathrm{const}$, only one modulus is a time operator, for example, the shear modulus G. We determine the form of the transform $\tilde{G}(s)$ for the operator G(t). We restrict ourselves to a Maxwell viscoelastic model [6]. Then the uniaxial compression of the dry porous stratum is described by

$$\frac{\partial\sigma_{ii}^f}{\partial t} + \frac{\sigma_{ii}^f}{\theta} = E\frac{\partial}{\partial t}\left(\frac{\partial u_i}{\partial x_i}\right), \tag{4.1}$$

where E is the instantaneous Young's modulus, η is viscosity, and $\theta = \eta/E$ is the matrix relaxation time. These parameters can be determined from creep experiments in one-dimensional compression, i.e., on rheological test of the type of (4.1).

We apply a Laplace transformation to (4.1) and use the initial condition $\sigma^f_{ii}(i, 0) = E\partial u_i(i, 0)/\partial x_i$ to get

$$\tilde{\sigma}^f_{ii}(s+1/\theta) = Es\partial u_i/\partial x_i. \tag{4.2}$$

Then for the operator $\tilde{E}(s)$ we get the expression

$$\tilde{E}(s)(1-m_0) = \tilde{\sigma}^f_{ii}/(\partial u_i/\partial x_i) = Es(s+1/\theta)^{-1}.$$

Consequently,

$$\tilde{G}(s)(1-m_0) = \tilde{E}(s)(1-m_0)/2(1+\nu) = Es[2(1+\nu)(s+1/\theta)]^{-1}. \tag{4.3}$$

From (2.16) and (4.3) we get the following expressions for the viscoelastic function $\tau'(\xi h)$ and the elastic one $\tau(\xi h)$:

$$\begin{gathered}\tau'(\xi h) = \tau(\xi h)(1+(s\theta)^{-1}),\\ \tau(\xi h) = \mu(1-2\nu)(1+\nu)(3/2-\nu)/kE(1-\nu)(1+\Phi(\xi h))^{-1}.\end{gathered} \tag{4.4}$$

We substitute (4.4) and (2.16) into the solution for the pressure field in an unrestricted elastic stratum of (2.14) to get the viscoelastic solution

$$\int_0^\infty \bar{p}^*(\xi, t)\, e^{-st}dt = \xi\rho\,(-Q_0\mu/2\pi kh)\sin(\xi\rho)/s\left(\tau(\xi h)\left(s+\theta^{-1}\right)+\xi^2\right). \tag{4.5}$$

Successive inversion of (4.5) in accordance with the rules for Laplace and Hankel transformations gives

$$p^*(r, t) = (-Q_0\mu/2\pi kh)\int_0^\infty \sin(\xi\rho)\left(1-\exp\left(-(\theta^{-1}+\xi^2/\tau(\xi h))\,t\right)\right)J_0(\xi r)\left(\rho\left(\tau(\xi h)/\theta+\xi^2\right)\right)^{-1}d\xi. \tag{4.6}$$

Similarly, for a closed stratum we get

$$\begin{gathered}p^*(r, t) = (-Q_0\mu/2\pi kh)\,2R^{-2}\sum_i \sin(\xi_i\rho)\left(1-\exp\left(-t\left(\theta^{-1}\right.\right.\right.\\ \left.\left.\left.+\xi_i^2/\tau(\xi_i h)\right)\right)\right)J_0(\xi_i r)/([J_0(R\xi_i)]^2\rho\,(\tau(\xi_i h)/\theta+\xi_i^2)).\end{gathered} \tag{4.7}$$

The solutions of (4.6) and (4.7) for $\tau(\xi h)\xi^2/\theta \to \infty$ go over to the solutions of (2.14) and (3.7) corresponding to an elastic stratum.

5. If we introduce the dimensionless quantities: $x = \xi\rho$, $h' = h/\rho$, $p' = -p_* 2\pi kh/Q_0\mu$, $\theta' = \tau(xh')\rho^2\theta^{-1}$, $T = t(1+\Phi(\xi h))/\tau\rho^2$, then from (2.14) we get the result for the average pressure in a window:

$$\langle p'\rangle = \int_0^\infty \sin x\,[1-\exp(-Tx^2(1+\Phi(xh')))]\,J_1(x)\,x^{-3}dx. \tag{5.1}$$

We average the viscoelastic solution of (4.6) over the area of the crack to get in dimensionless form that

$$\langle p'\rangle = \int_0^\infty \sin x\,[1-\exp(-T(\theta'+x^2(1+\Phi(xh'))))]\,J_1(x)\left(x\left(\theta'(1+\Phi(xh'))^{-1}+x^2\right)\right)^{-1}dx. \tag{5.2}$$

We introduce the following dimensionless quantities for a closed stratum: $x_i = \xi_i R$, $\rho' = \rho/R$, $h' = h/R$, $T = t(1+\Phi(\xi_i h))/\tau(\xi_i h)R^2$, $\theta' = \tau(x_i h)R^2\theta^{-1}$, $p' = -p_*(t)\,2\pi kh/Q_0\mu$.

We average (3.7) and (4.7) with respect to the area over the crack to get the elastic solution in dimensionless form:

$$\langle p'\rangle = \sum_{i=1}^\infty \sin(x_i\rho')(x_i\rho')^{-2}\left[1-\exp\left(-Tx_i^2(1+\Phi(x_i h'))\right)\right]J_1(x_i\rho')\,/[J_0(x_i)\,x_i]^2 \tag{5.3}$$

and the viscoelastic solution

$$\begin{gathered}\langle p'\rangle = \sum_{i=1}^\infty \sin(x_i\rho')\left[1-\exp\left(-T\left(x_i^2(1+\Phi(x_i h'))+\right.\right.\right.\\ \left.\left.\left.+\theta'\right)\right)\right](1+\Phi(x_i h'))(x_i\rho')^{-2}\left((1+\Phi(x_i h'))x_i^2+\theta'\right)^{-1}J_1(x_i\rho')\,[J_0(x_i)]^{-2}.\end{gathered} \tag{5.4}$$

The following values were used in (5.1) and (5.2) for a detailed calculation by computer: $\mu\alpha k^{-1} = 0.96$, $\gamma = 0.6$, $\theta' = 2 \times 10^{-3}(\rho)^2$, while the values of ρ and h' were varied. There is only a very weak dependence of the solution on h' in the range $0.01 \leqslant h' \leqslant 10$ (with the values stated for the other parmeters); the change is not more than 5% of the solution for $h' = 0.1$, $\lg T = 2$. Figure 2 gives curves relating the dimensionless pressure $\langle p' \rangle$ to the dimensionless time T. Curves 1 and 2 correspond to the solution (2.15) for the locally elastic case and (5.1) for the nonlocally elastic case for an infinite stratum, while curves 3 and 4 represent the viscoelastic solution of (5.2) with $\theta' = 0.002$ and 0.2. The mode of variation in $\langle p' \rangle$ with T shows that the time taken to reach the steady state and the value of the pressure p' in that state are dependent on the matrix relaxation time and on the relative dimensions of the window.

Figure 3 shows curves relating the pressure p' to the dimensionless time for a closed stratum. The solid lines I-III correspond to the elastic solution with the values $\rho' = 10^{-4}$; 10^{-3}; 10^{-2}. For comparison, we give two broken curves that correspond to the elastic local solution for $\Phi(\xi_i h) = 0$. In an elastic stratum, the process does not reach a steady state, and the effects of the contour of the stratum cause an unrestricted increase in the pressure $\langle p' \rangle$, which is necessary to maintain a constant flow rate. Curves 1-3 correspond to the viscoelastic solution of (5.4) with $\theta' = 2 \cdot 10^3$; $2 \cdot 10^5$; $2 \cdot 10^7$.

The dependence of the dimensionless pressure at the crack on the viscosity of the matrix and the radius of the crack in (5.4) is the same as that in (5.1) (viscoelastic solutions). The less the viscosity of the matrix and the greater the crack radius, the sooner the steady state sets in and the less the corresponding pressure change.

The choice of optimum window parameters and tapoff conditions is thus associated with the relaxation parameters of the matrix in the deformable stratum.

We are indebted to É. A. Bondarev for interest.

LITERATURE CITED

1. V. N. Nikolaevskii, K. S. Basinev, et al., Mechanics of Saturated Porous Media [in Russian], Nedra, Moscow (1970).
2. C. R. Fast, G. B. Holman, and R. Covlin, "The application of massive hydraulic fracturing to the tight muddy 'j' formation, Watteberg Field, Colorado," J. Petrol. Technol., 29, 10 (1977).
3. I. N. Sneddon, Fourier Series, Routledge and Kegan (1973).
4. E. Gray and G. B. Mathews, Bessel Functions and Their Applications in Physics and Mechanics [Russian translation], Inostr. Lit., Moscow (1955).
5. V. N. Nikolaevskii, "The influx of liquid to a horizontal crack in a stratum," Izv. Akad. Nauk SSSR, OTN, No. 7 (1958).
6. R. M. Christensen, Theory of Viscoelasticity: An Introduction, Academic Press (1970).
7. Yu. N. Rabotnov, Elements of the Inherited Mechanics of Solids [in Russian], Nauka, Moscow (1977).